Few Body Systems manuscript No.
(will be inserted by the editor)

A relativistic coupled-channel formalism for electromagnetic form factors of 2-body bound states

E. P. Biernat · W. H. Klink · W. Schweiger

Received: date / Accepted: date

Abstract We discuss a Poincaré invariant coupled-channel formalism which is based on the point-form of relativistic quantum mechanics. Electromagnetic scattering of an electron by a 2-body bound state is treated as a 2-channel problem for a Bakamjian-Thomas-type mass operator. In this way retardation effects in the photon-exchange interaction are fully taken into account. The electromagnetic current of the 2-body bound state is then extracted from the one-photon-exchange optical potential. As an application we calculate electromagnetic pion and deuteron form factors. Wrong cluster properties, inherent in the Bakamjian-Thomas framework, are seen to cause spurious (unphysical) contributions in the current. These are separated and eliminated in an unambiguous way such that one is left with a current that has all the desired properties.

Keywords Relativistic quantum mechanics · Few-body systems · Electromagnetic structure

The electromagnetic structure of a few-body bound state is usually encoded in Lorentz invariant functions, the electromagnetic form factors. The electromagnetic current of the bound-state can then be written as a sum of independent Lorentz covariants which are multiplied by these form factors. The form factors (in the space-like region) are functions of Mandelstam t ($\equiv$ momentum transfer squared $q^2 = -Q^2$). The theoretical challenge is to find an appropriate expression for the bound-state current in terms of the constituents currents. Due to binding effects it obviously cannot be a simple sum of the constituent currents [1]. Furthermore, the bound-state current must transform like a four vector under Lorentz transformations, it has to be conserved and the form factors should be normalized correctly at $Q^2 = 0$.

Presented at the 21st European Conference on Few-Body Problems in Physics, Salamanca, Spain, 30 August - 3 September 2010.

E. P. Biernat · W. Schweiger
Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria
E-mail: elmar.biernat@uni-graz.at, wolfgang.schweiger@uni-graz.at

W.H. Klink
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa, USA
E-mail: william-klink@uiowa.edu

We use the point form of relativistic quantum mechanics which is characterized by the property that only the four generators of space-time translations are interaction dependent. In order to introduce interactions into these generators in a Poincaré invariant manner we make use of the, so-called, "Bakamjian-Thomas construction" [2]. By means of this construction the (interacting) four-momentum operator becomes a product of an interacting mass operator times a free four-velocity operator. It thus suffices to study just the eigenvalue problem for the mass operator. In the point form it is most convenient to use velocity states as basis states. Velocity states describe the system by its overall velocity $\mathbf{v}$ and the center-of-mass momenta $\{\mathbf{k}_i\}$ and canonical spin projections $\{\mu_i\}$ [3] of its components (plus possible discrete quantum numbers). Notably, the Bakamjian-Thomas approach allows even for instantaneous interactions. Its drawback, however, is the violation of cluster separability (macroscopic locality) [4]. In the following we will focus on how this may affect electromagnetic currents and discuss a possible solution.

We treat elastic electron-bound-state scattering as a 2-channel problem for a Bakamjian-Thomas type mass operator and use the velocity-state representation. The emission and absorption of the photon (γ) by the electron (e) and the 2 constituents (c_1, c_2) of the bound state (C) are described by vertex interactions which are derived from quantum field theory [5]. Inherent in the Bakamjian-Thomas framework is the conservation of the total four-velocity of the system which must also be demanded at the γ-vertices. The binding force between the constituents is described by an instantaneous potential which is added to the operator for the free invariant mass of the ec_1c_2 and the $ec_1c_2\gamma$ channels. After elimination of the $ec_1c_2\gamma$ channel by means of a Feshbach reduction one ends up with the 1-γ-exchange optical potential $V_{\rm opt}$. The bound-state current $J^\mu(\mathbf{k}'_C,\mu'_j;\mathbf{k}_C,\mu_j)$ can then be extracted from on-shell matrix elements of $V_{\rm opt}$. $J^\mu(\mathbf{k}'_C,\mu'_j;\mathbf{k}_C,\mu_j)$ is essentially an integral over bound-state wave functions, constituents currents and Wigner D functions (see, e.g., Ref. [6]).

For the case of a spin-0 particle like, e.g., the pion (π) it can be shown that $J^\mu(\mathbf{k}'_\pi;\mathbf{k}_\pi)$ is conserved and that the components of $B_c(\mathbf{v})^\mu_\nu J^\nu(\mathbf{k}'_\pi;\mathbf{k}_\pi)$ transform like a four-vector under Lorentz transformations[1]. It is clear that the physical pion current should depend only on one covariant times the pion form factor f, with f being a function of Q^2. This is, however, not the case for $J^\mu(\mathbf{k}'_\pi;\mathbf{k}_\pi)$ derived in the way we did. We rather need two covariants and 2 form factors f and b:

$$J^\mu(\mathbf{k}'_\pi;\mathbf{k}_\pi) = f(Q^2,s)(k'_\pi+k_\pi)^\mu + b(Q^2,s)(k'_e+k_e)^\mu. \tag{1}$$

The additional (current conserving) covariant is the sum of the electron momenta. Note that both, the physical form factor $f(Q^2,s)$ and the spurious form factor $b(Q^2,s)$, exhibit also a dependence on Mandelstam s (or equivalently on $k=|\mathbf{k}_C|$). The reason for this s-dependence and the spurious contribution to the current is the violation of cluster separability in the Bakamjian-Thomas construction. A numerical analysis shows that for large s the form factor $f(Q^2,s)$ becomes independent of s and $b(Q^2,s)$ vanishes [7]. In the limit $s\to\infty$ one obtains a simple analytic expression for the pion form factor [6], which is equivalent with the standard front form result of Ref. [8].

For the case of a spin-1 system we consider the deuteron (D) as an example. Our deuteron current $J^\mu(\mathbf{k}'_D,\mu'_j;\mathbf{k}_D,\mu_j)$ can be decomposed into 11 independent Lorentz-covariants which are constructed from the polarization vectors $\epsilon^{*\mu}(k'_D,\mu'_j)$ and $\epsilon^\mu(k_D,\mu_j)$ and the 4-momenta $(k_D+k'_D)^\mu$, $(k_e+k'_e)^\mu$ and q^μ. Correspondingly,

[1] $B_c(\mathbf{v})$ is a canonical (rotationless) boost.

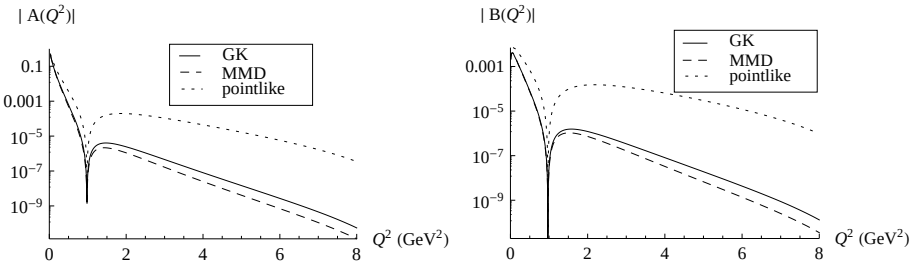

Fig. 1 Elastic scattering observables $A(Q^2)$ and $B(Q^2)$ for different parametrizations of the nucleon form factors (GK [12], MMD [13], point-like nucleons) for a simple Walecka-type model.

we get 3 physical form factors f_1, f_2, g_M and 8 spurious form factors. In the limit $k \to \infty$ the 3 physical form factors become again independent of k. Unlike the pion case not all spurious contributions vanish. This result resembles the one obtained within a covariant light-front approach [9]. But, most importantly, the physical form factors can be extracted from the current matrix elements in an unambiguous way. Using our standard kinematics with momentum transfer in 1-direction [6] we get:

$$\begin{aligned} F_1(Q^2) &= \lim_{k\to\infty} f_1(Q^2,k) = \lim_{k\to\infty} \frac{(-1)}{2k}\left[J^0(\mathbf{k}'_D,1;\mathbf{k}_D,1) + J^0(\mathbf{k}'_D,1;\mathbf{k}_D,-1)\right], \\ F_2(Q^2) &= \lim_{k\to\infty} f_2(Q^2,k) = \lim_{k\to\infty} \frac{(-2m_D^2)}{Q^2 k} J^0(\mathbf{k}'_D,1;\mathbf{k}_D,-1), \\ G_M(Q^2) &= \lim_{k\to\infty} g_M(Q^2,k) = \lim_{k\to\infty} \frac{(-\mathrm{i})}{Q} J^2(\mathbf{k}'_D,1;\mathbf{k}_D,1). \end{aligned} \tag{2}$$

The elastic scattering observables $A(Q^2)$ and $B(Q^2)$ for a simple Walecka-type model [10,11] are depicted in Fig. 1.[2]

At the end we want to emphasize that this kind of formalism is very general such that it is, e.g., also able to accommodate for n-particle bound states or for exchange-current effects which are caused by binding forces due to dynamical particle exchange.

Acknowledgements E.P.B. acknowledges the support of the "Fonds zur Förderung der wissenschaftlichen Forschung in Österreich" (FWF DK W1203-N16). He also thanks the province of Styria, Austria, for a PhD grant.

References

1. A. J. F. Siegert, Phys. Rev., 52, 787 (1937)
2. B. Bakamjian and L. H. Thomas, Phys. Rev., 92, 1300 (1953)
3. W. H. Klink, Phys. Rev. C, 58, 3617 (1998)
4. B. D. Keister, W. N. Polyzou, Adv. Nucl. Phys., 20, 225, (1991)
5. W. H. Klink, Nucl. Phys. A, 716, 123 (2003)
6. E. P. Biernat, W. Schweiger, K. Fuchsberger, W. H. Klink, Phys. Rev. C, 79, 055203 (2009)
7. E. P. Biernat, W. H. Klink, W. Schweiger, arXiv:1008.0244 [nucl-th]
8. P. L. Chung, F. Coester, W. N. Polyzou, Phys. Lett. B, 205, 545 (1988)
9. J. Carbonell, B. Desplanques, V. A. Karmanov, J. F. Mathiot, Phys. Rept., 300, 215 (1998)
10. J. D. Walecka, Ann. Phys., 83, 491 (1974)
11. For the model parameters see: B. L. G. Bakker and E. P. Biernat, in preparation.
12. M. Gari, W. Krumpelmann, Z. Phys. A. 322, 689-693 (1985)
13. P. Mergell, U. G. Meissner, D. Drechsel, Nucl. Phys. A, 596, 367-396 (1996)

[2] This calculation of deuteron form factors within a Walecka-type model is part of a benchmark calculation to fix the notion of "relativistic effects" in few-body problems.